Cambridge Assessment

A Level

Mathematics

Session: 1974 June
Type: Question paper
Code: 840

MATHEMATICS 1

840/1

ADVANCED LEVEL (SYLLABUS A)

(Three hours)

Answers to not more than **eight** *questions are to be given up.*

A pass mark can be obtained by good answers to about **four** *questions or their equivalent.*

Mathematical tables, a list of formulae, and squared paper are provided.

1 (*a*) Prove that

$$\log_y x = \frac{1}{\log_x y},$$

where $x > 0$ and $y > 0$. (6)

Find the possible values of x if

$$2(\log_9 x + \log_x 9) = 5.$$

(*b*) Solve the equation

$$\sqrt{(3x-5)} - \sqrt{(x+2)} = 1, \qquad (6)$$

where the positive values of the square roots are to be taken.

2 (*a*) In a given plane there are four given points on a straight line l and four other points, no three of which are collinear, and no two of which are collinear with any of the given points on l. Show that, in addition to l, 22 lines can be obtained by joining up the points in pairs.

Find the number of triangles that can be formed with vertices at three of the points. Make your method of calculation clear. (6)

(*b*) Show, by putting $a = b+c$ or otherwise, that $a-b-c$ is a factor of

$$a^4+b^4+c^4-2b^2c^2-2c^2a^2-2a^2b^2.$$

Factorise this expression completely. (6)

3 Find the values of a and b for which the first non-zero term in the expansion, in ascending powers of x, of the expression

$$\frac{1+ax}{1+bx}-e^x$$

is the one in x^3, and find the coefficient of x^3 in the expansion.

Deduce that, if x is small enough for powers of x above the second to be ignored,

$$e^x = \frac{2+x}{2-x}.$$

Using this approximation estimate the value of $e^{\frac{1}{20}}$ giving four places of decimals in your answer. (12)

4 (*a*) Prove that

$$\cos 3\theta - \sin 3\theta = (\cos\theta + \sin\theta)(1 - 4\cos\theta\sin\theta). \quad (4)$$

(*b*) Prove that if $\sec A = \cos B + \sin B$,

(i) $\tan^2 A = \sin 2B$,

(ii) $\cos 2A = \tan^2(\frac{1}{4}\pi - B)$. (8)

5 (*a*) Solve, for values of x between 0° and 360° inclusive, the equations

(i) $\cos 2x = \cos x$, (3)

(ii) $\sin x = 2\sin(60° - x)$. (4)

(*b*) If $p\cos 2x + q\sin 2x + r = 0$, where $p \neq 0$ and $p \neq r$, find an equation for $\tan x$.

Deduce that, if the roots of this equation are $\tan x_1$ and $\tan x_2$, then

$$\tan(x_1 + x_2) = \frac{q}{p}.$$ (5)

6 The vertices of triangle ABC are $A(-16, 0)$, $B(9, 0)$ and $C(0, 12)$. Prove that the equation of the internal bisector of the angle A of the triangle is $x - 3y + 16 = 0$. (4)

Find the equation of the internal bisector of the angle B of the triangle. (3)

Hence, or otherwise, find the equation of a circle which touches all three sides of the triangle. (5)

7 The tangents at $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ to the

parabola $y^2 = 4ax$, where $p > q$, meet at T. Find the coordinates of T in terms of a, p and q, and prove (in any order) that (3)

(i) the area of triangle PTQ is $\frac{1}{2}a^2(p-q)^3$, (4)

(ii) $\sin P\hat{T}Q = \dfrac{p-q}{\sqrt{\{(1+p^2)(1+q^2)\}}}$. (5)

8 Prove that the equation of the tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

at the point $(a\cos\theta, b\sin\theta)$ is (3)

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1.$$

If points P and Q on the ellipse have coordinates (4)

$$\big(a\cos(\theta+\alpha), b\sin(\theta+\alpha)\big) \quad \text{and} \quad \big(a\cos(\theta-\alpha), b\sin(\theta-\alpha)\big)$$

respectively, prove that the tangents at P and Q to the ellipse meet at the point T with coordinates

$$(a\cos\theta\sec\alpha, b\sin\theta\sec\alpha).$$

If M is the mid-point of PQ, prove that the line TM passes through the origin O. (5)

9 (*a*) If $f(x) = \ln(1+x) - x + \frac{1}{2}x^2$, find $f'(x)$.

Deduce that, if $x > 0$, then $f(x) > 0$. (6)

(*b*) A point P moves on the axis of x in such a way that, at time t, its displacement x from the origin O is given by

$$x = te^{-2t}, \quad \text{where } t \geqslant 0.$$

Find expressions for the velocity and acceleration of P at time t.

Hence show that for a finite interval of time (to be determined) the point P is moving towards O with a speed which increases with time. (6)

10 A cylinder is inscribed in a given right circular cone. The axis of the cylinder is along the axis of the cone and the base of the cylinder lies in the base of the cone. The circumference of the top of the cylinder lies in the curved surface of the cone. The base-radius of the cone is a and its semi-vertical angle is α. The base-radius of the cylinder may vary between 0 and a.

(i) Find the maximum volume of the cylinder. (6)

(ii) Given that $\cot\alpha = 3$ show that the *total* surface area of the cylinder is a maximum when its height and base-radius are equal. (6)

11 (*a*) Evaluate $\int_0^{\frac{1}{4}\pi} \sin 3\theta \sin\theta \, d\theta$. (4)

(*b*) Find $\int \sin^2\theta \cos^3\theta \, d\theta$. (4)

(*c*) Find $\int \frac{x}{\sqrt{(3+x)}} \, dx$. (4)

MATHEMATICS 2 **840/2**

ADVANCED LEVEL (SYLLABUS A)

(*Three hours*)

Candidates must not give up answers to more than **eight** *questions, of which not more than* **two** *may be chosen from Section 2.*

A pass mark can be obtained by good answers to about **four** *questions or their equivalent.*

Mathematical tables, a list of formulae, and squared paper are provided. In numerical work take g to be 9·8 m s^{-2}.

SECTION 1

Mechanics

1 A car starts from rest and moves in a straight line with constant acceleration $2f$ m s^{-2} for a time t s, and acceleration f m s^{-2} for a further $2t$ s. It then moves with constant velocity for a time $3t$ s. The engine is switched off and the brakes are applied to bring it to rest with a constant retardation $\frac{1}{2}f$ m s^{-2}. Draw a sketch of the velocity–time graph and determine the total distance travelled. (6)

Assuming that there is a constant resistance, of magnitude R N, throughout the motion and that the car has mass M kg, find the total work done by the engine and the energy dissipated by the brakes. (6)

2 A particle is projected from O with a velocity of magnitude $(27ga)^{\frac{1}{2}}$ so as to hit a target A which is at a horizontal distance $9a$ from O and at a height $6a$ above the level of O. If the two possible angles of projection are denoted by α_1, α_2, where $\alpha_1 > \alpha_2$, show that $\alpha_2 = 45°$. (4)

If the particle is projected at the larger angle, α_1, find the time of flight from O to A and the magnitude and direction of the velocity of the particle when it reaches A. (8)

3 A man cycles due north along a straight road with constant speed v. The wind appears to come from a direction $\theta°$ east of north. The man turns round and cycles due south at a constant speed $2v$ and finds that the wind now appears to come from a direction $\phi°$ east of south.

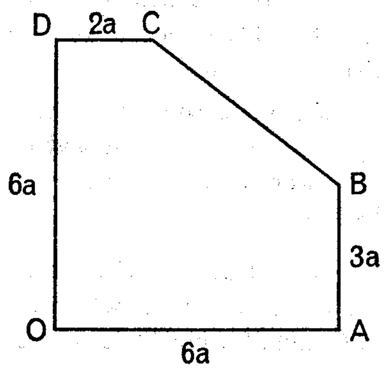
Show that the wind actually comes from a direction $\alpha°$ east of north, where

$$\tan\alpha = \frac{3\tan\theta\tan\phi}{2\tan\phi - \tan\theta}. \quad (12)$$

4 (*a*) A particle describes simple harmonic motion in a straight line. The particle is instantaneously at rest at time $t = 0$ and is next at rest at time $t = 3$ s. The speed at time $t = 2$ s is 1 m s^{-1}. Determine the amplitude of the motion and the maximum speed of the particle. (6)

(*b*) A particle of mass m moves in a horizontal circle with constant speed $\sqrt{(3ga/2)}$ on the smooth inside surface of a fixed sphere with centre O and radius a. Find the depth below O of the plane of the circle. (6)

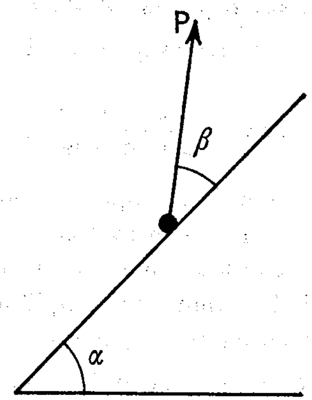
5



A piece of uniform wire of length $22a$ is bent into the shape shown in the figure, the angles at O, A, D being right angles. Find the coordinates of the centre of gravity referred to OA and OD as axes of x and y respectively. **(6)**

The wire is suspended from C and hangs in equilibrium with CB inclined at an angle α to the vertical. Find the value of $\tan\alpha$. **(6)**

6

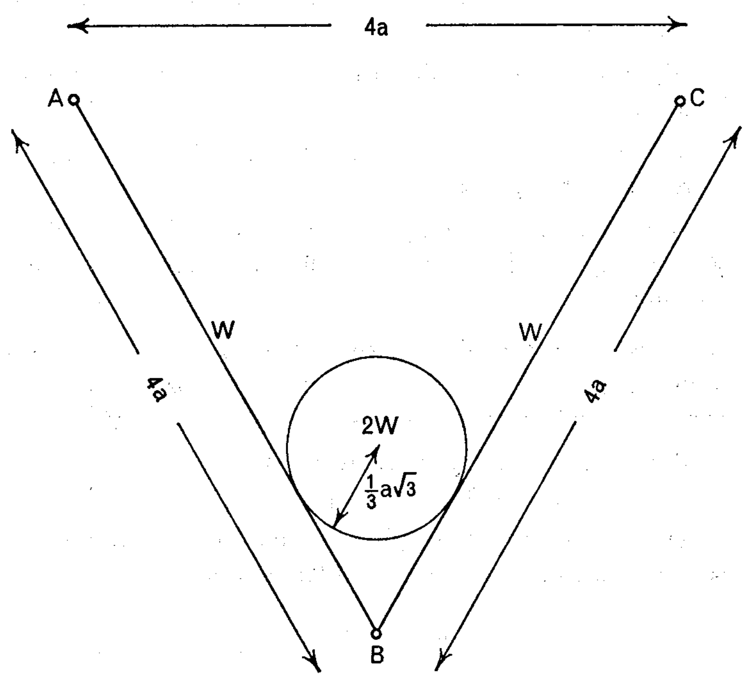


A particle of weight W is in contact with a rough plane, which is inclined at an angle α to the horizontal. The angle of friction is λ ($> \alpha$). A force of magnitude P acts on the particle in a vertical plane through a line of greatest slope and making an angle β with a line of greatest slope (see figure), and is just

sufficient to cause the particle to slide up the plane. Find an expression for P and show that the least possible value of P, as β varies, is $W\sin(\alpha+\lambda)$. **(7)**

If the corresponding least value of the magnitude of the force sufficient to make the particle move down the plane is $\frac{1}{3}W\sin(\alpha+\lambda)$, show that $\tan\lambda = 2\tan\alpha$. **(5)**

7



Two equal uniform rods AB and BC each of weight W and length $4a$ are smoothly hinged together at B. The ends A and C are smoothly hinged to two fixed points at the same horizontal level and at a distance $4a$ apart. A smooth uniform circular disc of weight $2W$ and radius $\frac{1}{3}a\sqrt{3}$ rests on the two rods (see figure) and is in equilibrium, the disc and the rods being in the same vertical plane. Determine the magnitude and direction of the reactions on the rod AB at A and B. **(12)**

Statistics

8 Eight trees are planted in a circle in random order. If two of the trees are diseased and later die, what is the probability that the two dead trees are next to each other? **(3)**

If four of them are diseased find (i) the probability that at least two of them are next to each other, and (ii) the probability that all four are next to each other. **(9)**

9 When a boy fires an air-rifle the probability that he hits the target is p.

(i) Find the probability that, firing 5 shots, he scores at least 4 hits. **(4)**

(ii) Find the probability that, firing n shots ($n \geqslant 2$), he scores at least two hits. **(3)**

(iii) Using a suitable approximation find the probability that, firing 400 shots, with $p = 0{\cdot}2$, he scores more than 100 hits. **(5)**

10 A random variable X takes values in $0 \leqslant x \leqslant 2\pi$ with probability density function $f(x) = a + b\cos x$. Find the value of a and show that $-1 \leqslant 2\pi b \leqslant 1$. **(4)**

Sketch the graph of $f(x)$ and hence state, with a reason, the expected value μ of X. **(3)**

Obtain the cumulative (distribution) function and hence, or otherwise, show that, if $b = 1/(2\pi)$ and $p(|X-\mu| < \alpha) = \frac{2}{3}$,

then $$\sin\alpha = \alpha - \frac{2\pi}{3}.$$ **(5)**

11 The mass of a coin has a distribution with mean 3·55 g. Taking the standard deviation to be 0·05 g and making a suitable assumption concerning the distribution of the mass of a coin, find limits within which the total mass of 100 coins will lie with 99 % probability. **(7)**

Find the largest possible value of the standard deviation σ if the probability that 100 coins have total mass inside the limits 354 g and 356 g is to exceed 99·9 %. **(5)**

12 (*a*) In a study of the number of matches in a box the contents of 100 boxes from one manufacturer were counted. Denoting the number of matches in a box by $50+x$ the results obtained gave $\Sigma x = -88$, $\Sigma x^2 = 481$. Making suitable assumptions (which should be stated) obtain a 95 % confidence interval for the average number of matches in a box. **(6)**

(*b*) Another manufacturer claims that, on average, 60 % of his matchboxes contain 50 or more matches. In a sample of 200 of the matchboxes, 59 % contained 50 or more matches. Test the manufacturer's claim statistically, against the hypothesis that less than 60 % of the boxes contain 50 or more matches. State any assumptions which you make and give adequate details of your test. **(6)**

13 The marks of seven university students in their A-level examinations, first year examinations and final examinations were as follows:

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*
A-level	57	82	67	53	62	74	48
First year	48	96	35	45	73	52	40
Final	35	75	Absent	45	65	55	30

Calculate a rank-correlation coefficient

(i) between A-level and first year examinations,

(ii) between A-level and final examinations,

and comment on the results. **(12)**

[The table below gives the least values of $|\rho|$ and $|\tau|$ which are significant at the 5 % level. **Either** Spearman's **or** Kendall's coefficient may be used.

Sample size r	Spearman's coefficient ρ	Kendall's coefficient τ
5	1	1
6	0·89	0·87
7	0·79	0·71
8	0·74	0·64
9	0·68	0·56
10	0·65	0·51

]

14 Two different types of sunshine recorder are used for a week and the following hours of sunshine recorded. **(12)**

Day	1	2	3	4	5	6	7
Recorder A	8·5	3·2	0·4	5·7	10·4	10·5	6·3
Recorder B	8·8	3·1	0·7	6·3	10·2	10·8	6·1

Making suitable assumptions (which should be stated) about the data, use a paired sample test to compare the data.

Give full details of your test and state your conclusions.

SECTION 2

Answer not more than **two** *questions from this Section.*

15 A surveyor at a point A observes that the summit M of a mountain is in a direction $\phi°$ east of north and that the angle of elevation of AM above the horizontal is $\alpha°$. He also observes that a point B is due north of A and that the angle of elevation of AB above the horizontal is $\theta°$. The surveyor moves to B and observes that M is now in a direction $\psi°$ east of north. He measures the distance AB and finds that $AB = l$. Show that the height of M above the horizontal level of A is

$$\frac{l\tan\alpha\sin\psi\cos\theta}{\sin(\psi-\phi)}. \qquad \textbf{(7)}$$

Show also that the angle of elevation $\beta°$ of BM above the horizontal is given by

$$\tan\beta = \operatorname{cosec}\phi\{\tan\alpha\sin\psi - \tan\theta\sin(\psi-\phi)\}. \qquad \textbf{(5)}$$

16 Show that the area of the region enclosed between the curve $y = -x^2+4$ and the straight line $y = -2x+4$ is $\frac{4}{3}$, and find the coordinates of the centroid of the region. **(5,7)**

17 (i) If $x = 2$ is a root of the equation

$$\alpha^2x^2+2(2\alpha-5)x+8 = 0$$

find the possible value (or values) of α and the corresponding value (or values) of the other root. **(5)**

(ii) Find the range (or ranges) of possible values of the real number α if

$$\alpha^2x^2+2(2\alpha-5)x+8 > 0$$

for all (real) values of x. **(7)**

[You may leave your answers in surd form.]

18 In a certain chemical reaction the differential equation relating the quantity x of a substance and the time t is

$$\frac{dx}{dt} = k(a-x)(b-x),$$

where k, a and b are positive constants, and $x = 0$ when $t = 0$. Obtain the solution for x as a function of t in the two cases (i) $a = b = 1$, (ii) $a = 1$, $b = 2$. **(5,7)**

MATHEMATICS **840/0**

SPECIAL PAPER (SYLLABUS A)

PAPER O

(Three hours)

Candidates may attempt as many questions as they please, but marks will be assessed on the **eight** *questions best answered.*

Mathematical tables, a list of formulae, and squared paper are provided.

1 Prove that

$$a^2+b^2+c^2-bc-ca-ab = \tfrac{1}{2}\{(b-c)^2+(c-a)^2+(a-b)^2\}. \qquad \textbf{(1)}$$

Given that

$$bc+ca+ab = k(a^2+b^2+c^2), \qquad \textbf{(4)}$$

prove that $k \leqslant 1$. Under what conditions is $k = 1$? **(1)**

If, further, a, b, c are the lengths of the sides of a triangle, prove that $k > \frac{1}{2}$. **(5)**

By considering triangles with $b = c = 1$, or otherwise, prove that it is possible to obtain triangles in which k is arbitrarily close to $\frac{1}{2}$. **(2)**

2 (*a*) Indicate in a diagram the points in the (x, y)-plane whose coordinates lie in the ranges

$$-2\pi \leqslant x \leqslant 2\pi, \quad -2\pi \leqslant y \leqslant 2\pi$$

and satisfy the equation

$$\sin^2 x + \cos^2 y = 2. \qquad (4)$$

(*b*) Find all the (real) values of x and y that satisfy the simultaneous equations

$$16\sin x + 9\tan y = 35,$$
$$64\sin^2 x + 27\tan^2 y = 259, \qquad (6)$$

where x, y are the measures of the angles expressed in degrees. (4)

3 Two of the points of intersection of the loci given by

$$x = 2k\cos t, \quad y = k\sin t \quad (k > 0),$$

and

$$x = cu, \quad y = c/u \quad (c > 0)$$

lie on the straight line $x = y$. Express c in terms of k. (6)

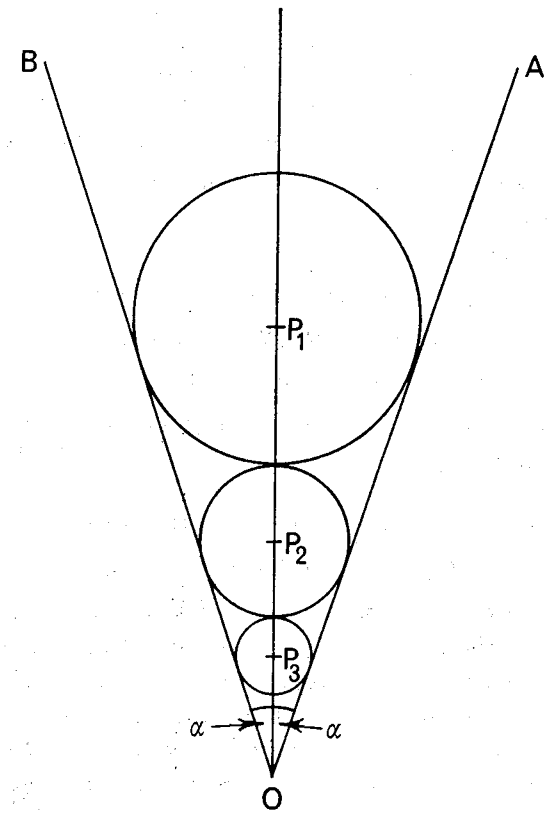
Prove that the four points of intersection of the two curves are at the vertices of a parallelogram whose sides have lengths k and $3k$. (7)

4 Find the coordinates of the points common to the curves

$$y = x^2 - 1, \qquad (4)$$
$$y = (x^2 - 1)^3. \qquad (2)$$

Sketch these curves in the same diagram. (7)

5



The diagram represents two straight lines OA, OB inclined at an angle 2α. The circle of centre P_1 has radius a and touches each of OA, OB. A sequence of circles is drawn, decreasing in radius, each touching OA, OB and its immediate predecessor. Prove that the areas of these circles are in geometric progression. (4)

The sum of the first n of these areas is S_n and the sum to infinity of the geometric progression is S. Prove that the difference $S - S_n$ is less than $\frac{1}{100}S$ whenever n exceeds

$$1/\lg\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right). \quad [\lg x \equiv \log_{10} x.] \qquad (6)$$

Prove also that the area of the first circle is equal to the sum of the areas of all the other circles when $\sin\alpha = 3-2\sqrt{2}$. (3)

6 Prove that the curves

$$y = a(e^{x/a} - 1),$$
$$y = a\ln\left(1 + \frac{x}{a}\right) \quad a > 0 \qquad \textbf{(3)}$$

touch at the origin O, and sketch them in the same diagram.

The line $x = k$ $(k > 0)$ meets the first curve at P, the second at Q and the line $y = x$ at R. Find the areas of the 'triangles' bounded by **(3)**

(i) the straight lines OR, RP and the arc OP,

(ii) the straight lines OR, RQ and the arc OQ, **(3)**

and prove that, as $k \to 0$, the ratio of the two areas tends to 1.

[The expansions of e^{ζ} and $\ln(1+\zeta)$ for small values of ζ may be assumed.] **(4)**

7 A student, asked to find a root of the equation

$$f(x) \equiv x^3 - 14x^2 + 49x - 8 = 0,$$

did not notice the solution $x = 8$ but chose, instead, to use Newton's method, taking $x = 7{\cdot}2$ as the first approximation. He then calculated, correctly, $f(7{\cdot}2) = -7{\cdot}712$, $f'(7{\cdot}2) = 2{\cdot}92$ and deduced (again correctly) the second approximation $9{\cdot}84$. By means of a graph, or otherwise, explain why Newton's method failed to give a better approximation in this case. **(4)**

Prove that, using Newton's method, a first approximation α, for a value of α in the interval $7{\cdot}2 < \alpha < 8$, would give a second approximation which is closer to the root $x = 8$ provided that

$$2(8-\alpha)f'(\alpha) + f(\alpha) > 0, \qquad \textbf{(4)}$$

and deduce that any value of α in the above range exceeding $5 + \sqrt{(5{\cdot}6)}$ would in fact give improvement. **(1)**

[You may, if you wish, assume without proof that $f'(x)$ is positive and increasing for $x > 7{\cdot}2$; also that **(4)**

$$2(8-\alpha)f'(\alpha) + f(\alpha) = (8-\alpha)(5\alpha^2 - 50\alpha + 97).]$$

8 Two beads, of masses m, M, are threaded on a fixed smooth circular wire of radius a whose plane is vertical. Initially the bead of mass m is held at rest at the level of the centre and the bead of mass M is at rest at the lowest point. The first of the beads is released and collides with the second, without loss of energy, and the velocities after impact, in the sense of motion of the first bead, are v, V respectively. Using the further condition of conservation of momentum, prove that

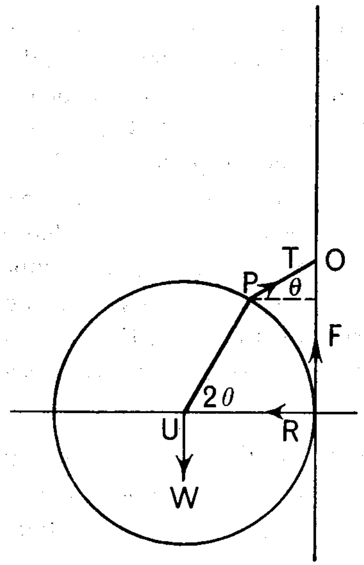
$$V - v = u,$$

where $u(= \sqrt{(2ga)})$ is the velocity of the first bead just before impact. **(3)**

Assuming that neither bead rises to the highest point of the wire, find the greatest height above the point of impact reached by each bead. **(5)**

If $M = 3m$, find the velocities immediately after the next impact, assuming energy still to be conserved. **(5)**

9



The diagram represents a uniform circular disc of radius a and weight W suspended in equilibrium against a vertical wall by a light inextensible string OP of length $a/\sqrt{3}$ attached to the disc at P, the plane of the disc being perpendicular to the wall. The radius UP makes an angle 2θ with the horizontal, and the string makes an angle θ with the horizontal. Find the value of θ. (3)

Find the least value of the coefficient of friction between the disc and the wall in order for equilibrium to be possible in the position shown, and find the tension in the string in this position. (6)

Alternatively, the disc is suspended in a similar way but so that OPU is a straight line. Prove that equilibrium is possible for any value of the coefficient of friction. (4)

10 A horizontal line A_1A_n is divided at points

$$A_2, A_3, \ldots, A_{n-1}$$

so that

$$A_1A_2 = a, \quad A_2A_3 = \tfrac{1}{2}a, \quad A_3A_4 = \tfrac{1}{3}a, \quad \ldots,$$

$$A_{n-1}A_n = \frac{1}{n-1}a.$$

Particles, each of mass m, are placed at $A_1, A_2, \ldots, A_n$. The particle at A_1 is projected with upward vertical component of velocity v and with horizontal component of velocity u such that, if unimpeded, it would meet A_1A_n produced at B, where $A_1B = (n-1)a$. Simultaneously the particles at $A_2, A_3, \ldots, A_n$ are projected vertically upwards with velocity v. Any collisions that take place are inelastic, so that colliding particles coalesce, and momentum may be assumed to be conserved both horizontally and vertically at impact. Determine the motion up to the instant when the particle initially at A_1 returns, with any adhering particles, to the level of the line A_1B. (13)

11 Five points A, U, H, V, B are in a vertical line, named in order downwards. The lengths of the segments are

$$AU = a, \quad UH = e, \quad HV = f, \quad VB = b,$$

and the points A, B are fixed. A light string of natural length a and modulus $mga\lambda$ joins A to a particle of mass m at H; a light string of natural length b and modulus $mgb\mu$ joins B to that particle. The system is then in equilibrium. Prove that

$$\lambda e - \mu f = 1. \quad (4)$$

A smooth bead of mass M is now threaded to the upper string and lies on top of the mass m at H. The mass M is not subject to the tension in the string. The system is released from rest. Prove that, during subsequent vibration while the two particles are still in contact and the strings are taut, the reaction between the particles is

$$Mm\{1+(\lambda+\mu)x\}g/(M+m) \quad (9)$$

when x is the distance of the particles below H.

12 Two pointers are free to move in a horizontal plane about their centres which are fixed. They are each spun and allowed to come to rest. The magnitudes of the angles Θ_1, Θ_2 which each makes with due north are observed ($0 \leqslant \Theta_1 < \pi$, $0 \leqslant \Theta_2 < \pi$). Determine the cumulative frequency (distribution) function of Θ_1 and of Θ_2. (3)

Hence, or otherwise, determine the cumulative frequency (distribution) function of X, where X is defined to be the larger of Θ_1 and Θ_2. Find the mean of X. (5,5)

13 A farmer wishes to compare the efficiency of two types of cattle food. He divides his herd of 20 cows into two groups of 10 and feeds the first group with food A and the second group with food B. After a reasonable time he measures, in gallons, the milk yield of each cow on a particular day, with the following results

First group (food A): $\Sigma x = 29{\cdot}5$, $\Sigma x^2 = 92{\cdot}11$.
Second group (food B): $\Sigma y = 33{\cdot}6$, $\Sigma y^2 = 116{\cdot}74$.

Making suitable assumptions (which should be stated), test the results statistically at the 5% level of significance. Explain what is meant by the phrase 'at the 5% level of significance'. (13)

14 An urn contains n balls labelled with the numbers $1, 2, \ldots, n$ respectively. A ball is chosen at random and if it carries the number 1 it is retained and not replaced; otherwise it is returned to the urn and the process is repeated. When the ball carrying the number 1 is obtained the process is repeated until the ball carrying the number 2 is obtained, and it is then retained and not replaced. The process is continued until all the balls have been removed. Find the probability distribution and the expected value of X_r, the number of selections needed to obtain the ball carrying the number r (the first $(r-1)$ balls having been removed). (4)

Find the expected number of selections needed to remove all n balls from the urn. (4)

[You may assume that $1+2x+3x^2+\ldots = (1-x)^{-2}$, $0 \leqslant x < 1$.]

(5)

15 The contents of a random sample of 24 bottles filled by an automatic bottling machine are measured and the following results obtained (in cm³). By introducing a suitable grouping of the data estimate the mean and variance of the population.

998·5	999·3	1002·4	1000·1	999·7	999·8	999·5	1001·7
997·2	999·5	1001·8	1000·2	1001·3	1000·3	997·8	1001·4
999·0	1001·4	1000·3	1001·0	997·6	1001·9	1002·0	998·2

Making suitable assumptions, which should be stated, about the distribution involved, draw up a quality control chart for means of samples of 4. The sample means of 25 samples of size 4 are given below. Plot them on the chart and comment on the quality of production.

1000·5	1000·8	999·8	999·4	998·4	998·8	999·9	1000·4
1000·4	999·5	998·8	1000·2	1001·7	999·4	1000·3	1000·0
997·5	999·8	1000·3	1000·5	1001·3	1000·7	998·9	1000·2
999·7							

(13)