Cambridge Assessment

O Level

Mathematics

Session: 1974 June
Type: Question paper
Code: 450

MATHEMATICS

ORDINARY LEVEL, SYLLABUS A

ARITHMETIC

(One hour and a half)

Mathematical tables and squared paper are provided.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

SECTION I. [70 marks]

Answer **all** *the questions in this section.*

You must not use mathematical tables in working Questions 1–4.

1 (*a*) Simplify

$$\frac{(\frac{5}{8} \times 7\frac{1}{5}) - (\frac{3}{5} \times 2\frac{11}{12})}{4\frac{1}{8}}.$$

(5)

(*b*) In a certain village $\frac{3}{16}$ of the inhabitants are over 60 years old, $\frac{1}{8}$ are between 15 and 30 years old and $\frac{1}{4}$ are under 15. The remaining 35 people are between 30 and 60 years old. If there are 7 men in the village who are over 60 find the number of women over 60. (5)

2 (*a*) The difference between two numbers is 3·9. The larger of the two is 14·7. Find the other number and the product of the two numbers. (5)

(*b*) Find the greatest number of books, each costing £1·65, which can be bought for £75. If the money left over is spent on pamphlets costing 5p each, find the number of pamphlets bought. (5)

3 (*a*) A sheet of postage stamps consists of 20 rows of stamps, with 10 stamps in each row, these being surrounded by an edging 9 mm wide. The whole sheet is 49·8 cm long and 43·3 cm wide. Calculate the area of each stamp, neglecting perforations. (7)

(*b*) A man takes 80 equal steps to walk from his front gate to a bus-stop. His son takes 75 steps in the opposite direction to reach his school. Given that the distance between the school and the bus-stop is 117 metres and that the length of the son's step is two-thirds that of his father, calculate the length of the man's step as a decimal of a metre. Given also that the man takes 45 seconds to walk to the bus-stop, calculate his walking speed in kilometres per hour. (9)

4 (*a*) A window is in the form of a rectangle $ABCD$ with a semi-circular top on AD as diameter. Given that $BC = 28$ cm and that the area of the rectangle is three times the area of the semicircle, calculate

(i) the length of AB,

(ii) the perimeter of the window.

[Take π to be $3\frac{1}{7}$.] (8)

(*b*) A man invests a sum of money on which he receives interest at 8% per annum. His interest for the first year is £140. He pays tax at 40p in the pound on the interest. Find how much of the interest he has left and express it as a percentage of the sum invested. (8)

5 (*a*) Use tables to calculate, correct to three significant figures, the value of

$$\frac{342{\cdot}6 \times (5{\cdot}729)^2}{(62{\cdot}81)^3}.$$ (7)

(*b*) A steel bar is 30 cm long and its cross-section is a square of side 1 cm. Its mass is 239·4 g. A spherical ball-bearing of the same material has a mass of 6·798 g. Calculate the radius of the ball-bearing, giving your answer in centimetres correct to two places of decimals. (10)

[Take π to be 3·142. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.]

SECTION II. [30 marks]

Answer **two** *questions in this section.*

6 The total charge made by a car-hire firm is made up of a fixed charge for each day for which the car is hired plus a charge for each kilometre travelled. The table gives the total cost of a day's hire when the given distances are travelled.

Distance (km)	250	400	750
Total cost (£)	6	7·80	12

Taking 2 cm to represent 100 km on one axis and to represent £2 on the other axis, draw a graph to show the total cost of a day's hire for distances from 0 to 750 km. Use your graph to find (4)

(i) the fixed daily charge, (3)

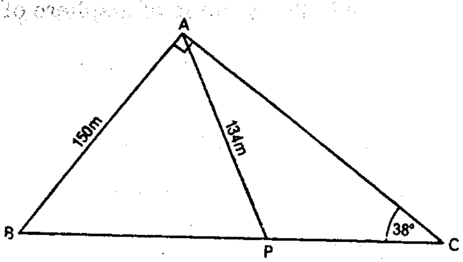
(ii) the total cost to a motorist who hires a car for two days and travels a total of 600 km. (5)

Another firm makes a fixed charge of £6 per day with no extra charge for the distance travelled. A motorist who hired a car for one day from the first firm finds that he could have saved £3 by hiring from the second firm. Find the distance he travelled. (3)

7 In a certain year the rateable value of a town was £3 200 000. Rates were collected at 80p for each pound of the rateable value. One eighth of the money so collected was allocated to road-works, the rest being divided between education and other services in the ratio 5:3. Find the amount allocated to education. (8)

In the following year the rateable value of the town was increased by 20% and the rates collected rose by $12\frac{1}{2}$%. Find the number of pence per pound of the new rateable value at which rates were charged. (7)

8



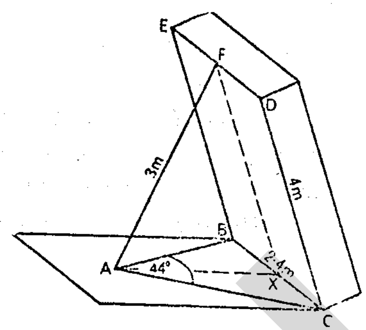
The diagram represents a triangular plot of ground ABC, situated at the junction of two straight roads BA and BC. Under a road-widening plan the section ABP is cut off. Given that $BA = 150$ m, $AP = 134$ m, $A\hat{C}B = 38°$ and $B\hat{A}C = 90°$, calculate

(i) AC, (3)

(ii) $A\hat{P}B$, (6)

(iii) the area of the remaining plot APC. (6)

9



The diagram represents the rectangular base $BCDE$ of a tipping trailer which has been raised from its horizontal position by the action of the extending bar AF. The horizontal bars AB and AC are equal. The points F and X are the midpoints of ED and BC respectively. Given that $DC = 4$ m, $BC = 2{\cdot}4$ m, $AF = 3$ m and $B\hat{A}C = 44°$, calculate

(i) AC, (4)

(ii) AX, (4)

(iii) the angle which $BCDE$ makes with the horizontal. (7)

MATHEMATICS 450/2

ORDINARY LEVEL, SYLLABUS A

GEOMETRY

(*Two hours*)

In questions involving calculations, no proofs are required but essential steps of the working must be shown.

SECTION I. [70 *marks*]

Answer **all** *the questions in this section.*

1 Without using set square or protractor, construct in a single diagram

(i) the triangle ABC such that $AB = 8$ cm, $A\hat{B}C = 60°$ and $BC = 6$ cm, (4)

(ii) the point D on the *opposite* side of AC to B such that $DA = DC$ and $BD = 9$ cm, (5)

(iii) the point P on AB such that PC bisects $B\hat{C}D$. (3)

2 (*a*) The sum of the interior angles of a convex polygon is five times the sum of the exterior angles. Calculate the number of sides of the polygon. (4)

(*b*) The diagonals AC and BD of the parallelogram $ABCD$ intersect at O. A line through O cuts AB and CD at P and Q respectively. Prove that

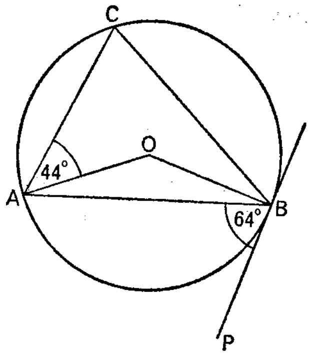
(i) the triangles AOP and COQ are congruent, (4)

(ii) the triangles ADP and CBQ are congruent. (6)

3 (*a*) A point R is 10 cm from the centre of a circle of radius 4 cm. A straight line RST cuts the circle at S and T. Given that $RS = 7$ cm, calculate ST. (4)

(b) In the acute-angled triangle XYZ, the point W is the foot of the perpendicular from X to YZ. Given that $XY = 7$ cm, $YZ = 10$ cm and $ZX = 9$ cm, calculate YW. (4)

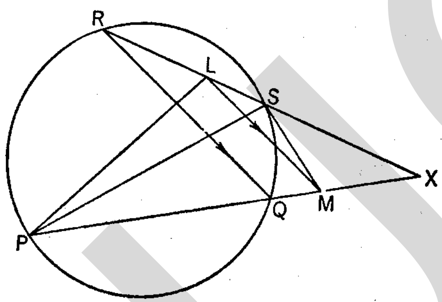
4 (a)



In the above figure, PB is the tangent to the circle at B. The centre of the circle is O. Given that $P\hat{B}A = 64°$ and $O\hat{A}C = 44°$, calculate $O\hat{A}B$ and $O\hat{B}C$. (6)

(b) Two circles of radii 9 cm and 4 cm touch externally. Calculate the length of the exterior common tangent. (4)

5



In the above figure, PQ and RS are chords of the circle which meet, when produced, at X. Points L and M, on RX and PX respectively, are such that LM is parallel to RQ. Prove that

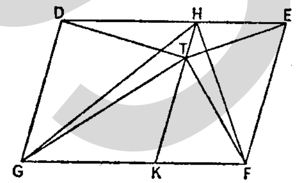
(i) $S\hat{L}M = S\hat{P}M$, (6)

(ii) $SLPM$ is a cyclic quadrilateral, (1)

(iii) the triangles XSM and XPL are similar, (3)

(iv) $PL.XM = SM.XL$. (2)

6 (a)



In the above figure, $DEFG$ is a parallelogram and the straight line HTK is parallel to DG. Prove that

Area $\triangle TDG$ + Area $\triangle TEF$ = Area $\triangle GHF$. (8)

(b) The point M is the mid-point of the chord AB of a circle, centre O, and S is a point on OM between O and M. Prove that $AS = SB$. (5)

Section II. [30 marks]

Answer **two** *questions in this section.*

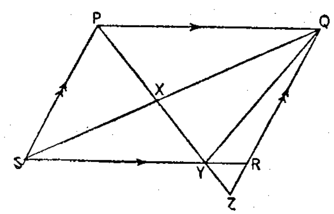
7 Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle. (6)

The point D is the mid-point of the side BC of triangle ABC. The bisector of $A\hat{D}B$ meets AB at X. The line through X parallel to BC meets AC at Y. Prove that

(i) YD bisects $A\hat{D}C$, (6)

(ii) XY is a diameter of the circle XDY. (3)

8



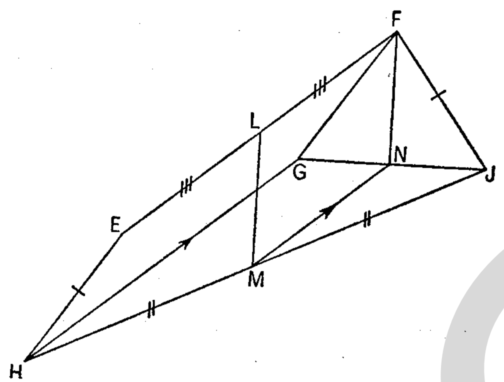
In the above figure, $PQRS$ is a parallelogram. A line through P cuts QS, RS, and QR produced at X, Y and Z. Prove that

(i) the triangles PXS and QXY are equal in area, (4)

(ii) $\dfrac{\text{Area } \triangle PXS}{\text{Area } \triangle QXZ} = \dfrac{XY}{XZ}$, (4)

(iii) $\dfrac{XY}{XZ} = \dfrac{PS^2}{QZ^2}$. (7)

9 Prove that the straight line drawn through the middle point of one side of a triangle parallel to another side bisects the third side. (6)



In the above figure, $EFGH$ is a parallelogram and $FJ = EH$. The mid-points of EF and HJ are L and M. The line through M parallel to HG meets GJ at N.

Prove that

(i) $G\hat{F}N = J\hat{F}N$, (6)

(ii) $FLMN$ is a parallelogram. (4)

10 Without using set square or protractor, construct in a single diagram

(i) a circle of radius 4 cm,

(ii) a triangle TPX where TP is the tangent to the circle at a point P, $TP = 3$ cm, $T\hat{P}X$ is obtuse and PX is a chord of the circle such that $PX = 7$ cm. (6)

(iii) the point Y on the circumference of the circle such that $P\hat{X}Y = 30°$ and $X\hat{Y}P = T\hat{P}X$, (4)

(iv) the point Z on YX produced such that the triangle PZY is equal in area to the quadrilateral $TXYP$. (5)

MATHEMATICS 450/3

ORDINARY LEVEL, SYLLABUS A

ALGEBRA

(One hour and a half)

Mathematical tables and squared paper are provided.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

SECTION I. [70 marks]

Answer **all** *the questions in this section.*

1 (*a*) Given that $a = -2$, $b = 1$, find the value of

$$2-3(a+2b)^2 \quad \text{and of} \quad \frac{a^2+b^2}{(a-b)^2}. \qquad (4)$$

(*b*) Express

$$2-\frac{5x}{x-1}+\frac{3x}{x+1} \qquad (4)$$

as a single fraction in its simplest form.

2 (*a*) Solve the equation $0{\cdot}2(x-3)-0{\cdot}5(x-2) = 1$. (4)

(*b*) Solve the simultaneous equations

$$4x+7y-5 = 5x-4y-13 = x+2y. \qquad (6)$$

3 Factorise completely

(i) $3+5k-2k^2$, (3)

(ii) $x(x-2)-y(2-x)$, (3)

(iii) $25-4(a+2)^2$. (3)

4 (*a*) Solve the equations

(i) $(2x+3)^2 = 9$, (3)

(ii) $(y+1)(y-1) = 3y+3$. (3)

(*b*) In an examination a boy gains p marks for each correct answer and loses q marks for each incorrect answer. He answers N questions altogether, of which x are found to be incorrectly answered, and the rest correctly answered. Find an expression for the total number of marks he obtains.

If $p = 3$, $q = 2$, $N = 25$, and his total mark is zero, calculate the value of x. (5)

5 (*a*) Solve the equation

$$x = \frac{1}{2x+3},$$ (7)

giving your answers correct to two places of decimals.

(*b*) Given that $(1-mk)t = m+k$, find an expression for m in terms of k and t. (4)

6 If A gives £5 to B then B will have four times as much money as A, but if, instead, B gives £9 to A then A will have twice as much money as B. Find how much money each had originally. (7)

7 (*a*) Using tables, calculate the value of the **cube root** of

$$\frac{uv}{u+v}$$

given that $u = 0{\cdot}234$ and $v = 5{\cdot}789$. Give your answer correct to three significant figures. (7)

(*b*) Given that $x = \frac{1}{2}$ and $y = 4$, calculate, without using any tables, the values of

(i) $x^{-y}+y^{-x}$, (3)

(ii) $\lg(10y)-\lg(8x)$. (3)

$$[\lg a = \log_{10} a]$$

SECTION II. [30 marks]

Answer **two** *questions in this section.*

8 (*a*) Solve the simultaneous equations

$$x-2y = 3,$$

$$\frac{1}{x}-\frac{2}{y} = 4\tfrac{1}{2}.$$ (8)

(*b*) Given that the remainder is 12 when the expression $ax^3+3x^2-11x-6$ is divided by $x+2$, calculate the value of a. Using this value of a, find out whether $2x+1$ is, or is not, a factor of the expression, showing all necessary working. (7)

9 (*a*) The sag in a plank supported at its two ends is found to vary directly as the cube of its length and inversely as the square of its thickness. A plank 10 metres long and 2 cm thick is found to sag 2·5 cm. Calculate the sag in a similar plank 15 metres long and 3 cm thick. (7)

(*b*) The first and last terms of an A.P. are -10 and 410 respectively. The sum of all the terms of the progression is 7200. Calculate the number of terms, and the common difference, of the progression. (8)

10 Two concerts were held. Tickets for the first concert cost x pence each and the total receipts from the tickets sold were £450. For the second concert, when the price of each ticket was reduced by 5 pence, 300 more tickets were sold and the total receipts increased by £90. Obtain an equation for x. Hence find

(i) the price of a ticket for the first concert,

(ii) the number of tickets sold for the first concert. (15)

11 The following is an incomplete table of values for the graph of $y = +\sqrt{25-4x^2}$.

x	-2	$-1\frac{1}{2}$	-1	0	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
y	3		4·58		4·58		3	

Calculate, and write down on your answer sheet, the missing values of y. (8)

Taking 2 cm as the unit for both x and y, draw the graph of $y = +\sqrt{25-4x^2}$ for values of x between -2 and $+2\frac{1}{2}$.

(i) From your graph estimate the range of positive values of x for which y lies between 3 and 4·5. (3)

(ii) By drawing another suitable graph on the same axes, estimate the value of the x-coordinate of the point P on the original graph such that the y-coordinate of P is twice the x-coordinate of P. (4)