Cambridge Assessment

A Level

Mathematics

Session: 1994 June
Type: Question paper
Code: 9205

UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

General Certificate of Education Advanced Level

MATHEMATICS (SYLLABUS C) 9205/1

PAPER 1

Wednesday **8 JUNE 1994** Morning 3 hours

Additional materials:
- Answer paper
- Formulae (List MF6)
- Graph paper (2 sheets)
- Mathematical tables

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions in Section I and any **four** questions from Section II.

Write your answers on the separate answer paper provided.

Begin each answer in Section II on a fresh page.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The use of an electronic calculator is expected, where appropriate.

This question paper consists of 7 printed pages and 1 blank page.

Section I

Answer **all** the questions in this section.

1 Given that $(2x+1)$ is a factor of $2x^3+ax^2+16x+6$, show that $a=9$. [2]

Find the real quadratic factor of $2x^3+9x^2+16x+6$. By completing the square, or otherwise, show that this quadratic factor is positive for all real values of x. [4]

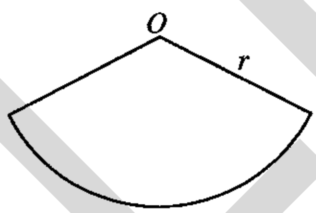
2 Expand $(1+2x)^{\frac{1}{4}}$, where $|x|<\frac{1}{2}$, as a series of ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 Given the simultaneous equations

$$2^x = 3^y, \quad x+y=1,$$

show that $x = \dfrac{\ln 3}{\ln 6}$. [4]

4



The diagram shows a sector of a circle, with centre O and radius r. The length of the arc is equal to half the perimeter of the sector. Find the area of the sector in terms of r. [3]

5 In triangle ABC, the lengths of the sides AB, BC and CA are 7 cm, 2 cm and b cm respectively, and the size of angle C is 30°. Use the cosine formula to show that $b^2-(2\sqrt{3})b-45=0$, and hence find the exact value of b. [3]

Hence, or otherwise, show that $\sin B = \dfrac{5\sqrt{3}}{14}$. [2]

6 Express $3\cos\theta - 5\sin\theta$ in the form $R\cos(\theta+\alpha)$, where $R>0$ and $0<\alpha<90°$. [2]

Hence, or otherwise, find the general solution of the equation $3\cos\theta-5\sin\theta=2$, giving your answer correct to the nearest 0·1°. [4]

7 The parametric equations of a curve C are

$$x = t+e^{t}, \quad y = t+e^{-t}.$$

Find $\dfrac{dy}{dx}$ in terms of t, and hence find the coordinates of the stationary point of C. [5]

8 The distinct points $P(\theta, \sin\theta)$ and $Q(\phi, \sin\phi)$ lie on the curve $y = \sin x$, where x is measured in radians. Show that the gradient of the chord PQ may be expressed as

$$\frac{\sin\frac{1}{2}(\phi - \theta)\cos\frac{1}{2}(\phi + \theta)}{\frac{1}{2}(\phi - \theta)}.$$ [2]

Deduce from this expression, explaining your working clearly, that if ϕ is approximately equal to θ then the gradient of PQ is approximately equal to $\cos\theta$. [2]

9 Express $\dfrac{1}{x^2(x-1)}$ in the form

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1},$$

where A, B and C are constants. [3]

Hence find $\displaystyle\int \frac{1}{x^2(x-1)}\,dx$. [2]

10 A set of 20 students is made up of 10 students from each of two different year-groups. Five students are to be selected from the set, and the order of selection is unimportant. Find

(i) the total number of possible selections, [1]

(ii) the number of selections in which there are at least two students from each of the two year-groups. [2]

11 **(i)** Show that

$$(k+1)^2\{2(k+1)^2 - 1\} = 2k^4 + 8k^3 + 11k^2 + 6k + 1$$

for all values of k. [2]

(ii) Prove by induction that

$$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2 - 1)$$

for all positive integers n. [3]

Section II

Answer any **four** questions from this section. Each question in this section carries 12 marks.

12 **(a)** A geometric progression has non-zero first term a and common ratio r, where $0 < r < 1$. Given that the sum of the first 8 terms of the progression is equal to half the sum to infinity, find the value of r, correct to 3 decimal places. [3]

Given also that the 17th term of the progression is 10, find a. [2]

(b) An arithmetic progression has first term a and common difference 10. The sum of the first n terms of the progression is 10 000. Express a in terms of n, and show that the nth term of the progression is

$$\frac{10\,000}{n} + 5(n - 1).$$ [3]

Given that the nth term is less than 500, show that $n^2 - 101n + 2000 < 0$, and hence find the largest possible value of n. [4]

13 **(a)** Functions g and h are defined by

$$\begin{aligned} &\mathrm{g} : x \mapsto \ln x, \quad && x \in \mathbb{R},\ x > 0, \\ &\mathrm{h} : x \mapsto 1 + x, \quad && x \in \mathbb{R}. \end{aligned}$$

The function f is defined by

$$\mathrm{f} : x \mapsto \mathrm{gh}(x), \quad x \in \mathbb{R},\ x > -1.$$

(i) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(ii) Write down expressions for $\mathrm{g}^{-1}(x)$ and $\mathrm{h}^{-1}(x)$. [2]

(iii) Write down an expression for $\mathrm{g}^{-1}\mathrm{h}^{-1}(x)$. [1]

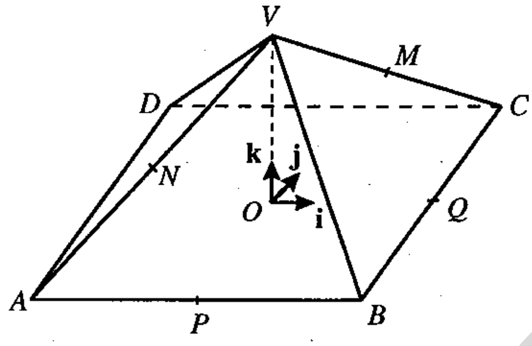
(iv) Sketch the graph of $y = \mathrm{g}^{-1}\mathrm{h}^{-1}(x)$. [2]

(b) The function q is defined by

$$\mathrm{q} : x \mapsto x^2 - 4x, \quad x \in \mathbb{R},\ |x| \leqslant 1.$$

Show, by means of a graphical argument or otherwise, that q is one-one, and find an expression for $\mathrm{q}^{-1}(x)$. [5]

14



In the diagram, O is the centre of the square base $ABCD$ of a right pyramid, vertex V. Perpendicular unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are parallel to AB, AD, OV respectively. The length of AB is 4 units and the length of OV is $2h$ units. P, Q, M and N are the mid-points of AB, BC, CV and VA respectively. The point O is taken as the origin for position vectors.

(i) Show that the equation of the line PM may be expressed as

$$\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ h \end{pmatrix},$$

where t is a parameter. [2]

(ii) Find an equation for the line QN. [2]

(iii) Show that the lines PM and QN intersect, and that the position vector $\overrightarrow{OX}$ of their point of intersection is $\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2}h \end{pmatrix}$. [3]

(iv) Given that OX is perpendicular to VB, find the value of h and calculate the acute angle between PM and QN, giving your answer correct to the nearest 0·1°. [5]

15 Given that $y = \cos(x + \alpha)\cos^2 x$, where α is a constant, show that, when $\frac{dy}{dx} = 0$, either $\cos x = 0$ or $\tan(x + \alpha) + 2\tan x = 0$. [5]

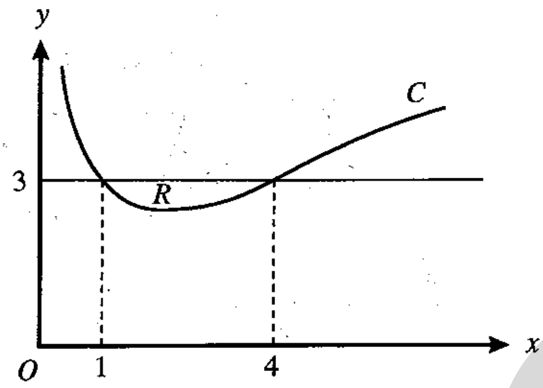
Given also that $\tan\alpha = \sqrt{2}$, with $0 < \alpha < \frac{1}{2}\pi$,

(i) find the exact values of $\tan x$ for which $\tan(x + \alpha) + 2\tan x = 0$, [4]

(ii) show that the value of y when $x = \alpha$ is exactly $-\frac{1}{9}$. [3]

[Turn over

16



The diagram shows the region R in the first quadrant bounded by the curve C with equation $y = \sqrt{x} + \dfrac{2}{\sqrt{x}}$ and the line $y = 3$. The line and the curve intersect at the points $(1, 3)$ and $(4, 3)$. Calculate the exact area of R. [4]

Write down the equation of the curve obtained when C is translated by 3 units in the negative y-direction. [2]

Hence, or otherwise, show that the volume of the solid formed when R is rotated completely about the line $y = 3$ is given by

$$\pi \int_1^4 \left(x - 6\sqrt{x} + 13 - \frac{12}{\sqrt{x}} + \frac{4}{x} \right) \mathrm{d}x,$$

and evaluate this integral exactly. [6]

17 **(i)** Show that

$$\int \sin^2 x \, \mathrm{d}x = \tfrac{1}{2}x - \tfrac{1}{4}\sin 2x + c,$$

where c is an arbitrary constant. [2]

(ii) Show that

$$\int_0^{\frac{1}{2}\pi} x \sin^2 x \, \mathrm{d}x = \tfrac{1}{16}(\pi^2 + 4).$$ [3]

(iii) By means of the substitution $2u = \cos x$, find the exact value of

$$\int_0^{\frac{1}{2}} \sqrt{(1 - 4u^2)} \, \mathrm{d}u.$$ [3]

(iv) By means of the substitution $2u = \cos x$, show that

$$\int_0^{\frac{1}{2}} 4u^2 \sqrt{(1 - 4u^2)} \, \mathrm{d}u = \tfrac{1}{8} \int_0^{\frac{1}{2}\pi} \sin^2 2x \, \mathrm{d}x,$$

and hence find the exact value of $\int_0^{\frac{1}{2}} 4u^2 \sqrt{(1 - 4u^2)} \, \mathrm{d}u$. [4]

18 It is given that, at any point on the graph of $y = \mathrm{f}(x)$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{(1 + y^3)}$.

(i) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{3}{2}y^2$. [3]

(ii) Find expressions for $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$ and $\dfrac{\mathrm{d}^4y}{\mathrm{d}x^4}$ in terms of y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(iii) The graph of $y = \mathrm{f}(x)$ passes through the origin. Show that, providing x is sufficiently small for powers of x higher than x^4 to be neglected,

$$y = x + \tfrac{1}{8}x^4.$$ [3]

The value of x at the point on the curve where $y = 0{\cdot}4$ is given by the integral

$$\int_0^{0\cdot4} \frac{1}{\sqrt{(1 + y^3)}}\,\mathrm{d}y.$$

Use the trapezium rule, with four intervals each of width $0{\cdot}1$, to find an approximate value for x where $y = 0{\cdot}4$, giving 3 decimal places in your answer. [3]

UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level

MATHEMATICS (SYLLABUS C) **9205/2**

PAPER 2

Tuesday **14 JUNE 1994** Morning 3 hours

Additional materials:
- Answer paper
- Formulae (List MF6)
- Graph paper (2 sheets)
- Mathematical tables

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **seven** questions.

Answer no more than **four** questions from any one option.

Write your answers on the separate answer paper provided.

Begin each answer on a fresh page.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

Each question carries 14 marks.

The number of marks is given in brackets [ ] at the end of each question or part question.

The use of an electronic calculator is expected, where appropriate.

This question paper consists of 8 printed pages.

Option (a): Particle Mechanics

1 [In this question take the value of g to be $10\,\text{m}\,\text{s}^{-2}$. Give answers for angles correct to the nearest $0{\cdot}1°$, and for forces correct to three significant figures.]

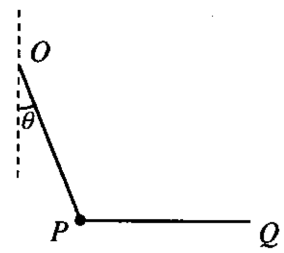


Fig. 1

A light inextensible string has one end attached to a fixed point O. To the other end is attached a particle P of mass $0{\cdot}5$ kg. A light inextensible string PQ is attached to P. The end Q is held so that P hangs in equilibrium with PQ horizontal (see Fig. 1). The tension in PQ is $1{\cdot}5$ N. Find the inclination θ of the string OP to the vertical, and find also the tension in OP. [6]

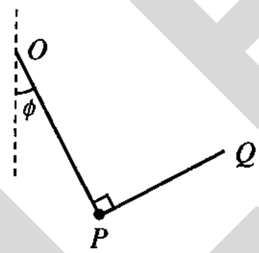


Fig. 2

The end Q is now moved and held so that P hangs in equilibrium, in a new position, with OP perpendicular to PQ (see Fig. 2). The tension in PQ is now 2 N. Find the inclination ϕ of OP to the vertical and find also the tension in OP. [5]

Given that OP has length $0{\cdot}3$ m, find the work done against gravity in slowly moving P from the first equilibrium position to the second, giving two significant figures in your answer. [3]

2 [In this question take the value of g to be $10\,\mathrm{m\,s^{-2}}$.]

At time $t = 0$ a particle P of mass 0·2 kg is projected from a point O and moves freely under gravity. The velocity of projection has horizontal component of magnitude $20\,\mathrm{m\,s^{-1}}$ and vertically upward component of magnitude $30\,\mathrm{m\,s^{-1}}$. Write down expressions for the horizontal and vertical displacements of P at time t seconds and hence show that the angle ϕ between OP and the upward vertical at O, at time t seconds, is given by

$$\tan\phi = \frac{4}{6-t}.$$ [4]

At the point A on the path of the particle, $\tan\phi = \frac{4}{3}$. Show that $OA = 75$ m, and find the velocity of the particle at A, giving its magnitude and direction. [5]

The particle passes through the point B when $t = 4·5$. Find the kinetic energy of the particle at this instant. [3]

Find also the angle between the tangent to the path of the particle at B and the horizontal. [2]

3 A particle P of mass 0·1 kg moves in a horizontal straight line under the action of a force of magnitude $\frac{1}{5x^2}$ newtons, directed towards a fixed point O of the line, where x metres is the displacement of P from O at time t seconds. At time $t = 0$, P passes through the point $x = 1$ with speed $2\,\mathrm{m\,s^{-1}}$, in the direction of increasing x. Write down a differential equation relating the speed $v\,\mathrm{m\,s^{-1}}$ of P to the displacement x metres, and hence show that

$$v = \frac{2}{\sqrt{x}}.$$ [5]

Find the loss in kinetic energy of P as it moves from $x = 1$ to $x = 9$. [2]

Find the time taken by P in moving from $x = 1$ to $x = 9$. [5]

Justifying your answer, state whether P ever returns to the point $x = 1$. [2]

4 A particle P of mass 0·2 kg is moving on a smooth horizontal table with speed $9\,\mathrm{m\,s^{-1}}$. A particle Q of mass 0·3 kg is moving on the table with speed $v\,\mathrm{m\,s^{-1}}$. The particles move directly towards each other and collide. In the collision the direction of motion of each particle is reversed. After the collision the speed of P is $3\,\mathrm{m\,s^{-1}}$ and the speed of Q is $2\,\mathrm{m\,s^{-1}}$. Find the magnitude of the impulse acting on P in the collision and find also the value of v. [4]

Find the total kinetic energy lost in the collision. [2]

The collision takes place at a distance 6 m from a fixed vertical wall whose plane is perpendicular to the line of motion of the particles. After the collision between P and Q, P is moving directly towards the wall. Show that when P strikes the wall, the distance between P and Q is 10 m. [3]

When P strikes the wall its velocity is reversed in direction but unchanged in magnitude. Find the distance from the wall at which the particles collide again. [3]

Find the total momentum of the system after the second collision between the particles. [2]

S94/33

UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE

General Certificate of Education Advanced Level

MATHEMATICS (SYLLABUS C) **9205/2**

PAPER 2

Tuesday **14 JUNE 1994** Morning 3 hours

ERRATUM NOTICE

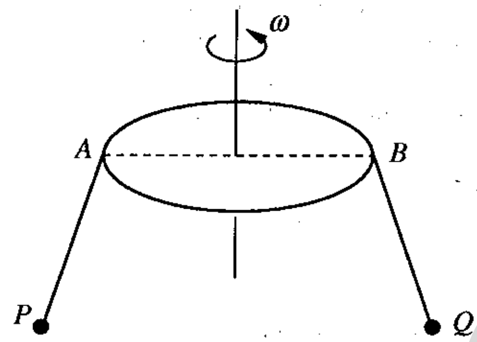
To be read to candidates

Page **4**, Question **5**, Paragraph 2.

The first sentence of the second paragraph should read:

The particles P and Q are now also connected by a horizontal light elastic string of natural length 6 m and modulus λ newtons.

5 [In this question take the value of g to be $10\,\mathrm{m\,s^{-2}}$.]



A circular disc of radius 1 m can be made to rotate in a horizontal plane about a fixed vertical axis passing through its centre. Points A and B are on the circumference of the disc at opposite ends of a diameter. A light inextensible string, of length 5 m, has one end attached to A and has a particle P of mass 0·5 kg attached to the other end. An identical string carrying an identical particle Q is attached to B. The disc is made to rotate and the strings remain in a vertical plane which rotates with constant angular speed $\omega\,\mathrm{rad\,s^{-1}}$ about the vertical axis (see diagram). Given that both strings remain at a constant angle of $\tan^{-1}(\frac{3}{4})$ to the vertical, find ω, correct to 3 significant figures. [6]

The particles P and Q are now connected by a light elastic string of natural length 6 m and modulus λ newtons. The disc is made to rotate and the strings remain in a vertical plane which rotates with constant angular speed $2\,\mathrm{rad\,s^{-1}}$ about the vertical axis. The two particles move in a horizontal circle of radius 5 m. Find λ. [8]

Option (b): Probability and Statistics

6 In a lottery there are 24 prizes allocated at random to 24 prize-winners. Ann, Ben and Cal are three of the prize-winners. Of the prizes, 4 are cars, 8 are bicycles and 12 are watches. Show that the probability that Ann gets a car and Ben gets either a bicycle or a watch is $\frac{10}{69}$. [2]

Giving each answer either as a fraction or as a decimal correct to 3 significant figures, find

(i) the probability that both Ann and Ben get cars, given that Cal gets a car, [3]

(ii) the probability that either Ann or Cal (or both) gets a car, [3]

(iii) the probability that Ann gets a car and Ben gets either a car or a bicycle, [3]

(iv) the probability that Ann gets a car given that Ben gets either a car or a bicycle. [3]

7 The continuous random variable X is the distance, measured in hundreds of kilometres, that a particular car will travel on a full tank of petrol. It is given that

$$P(X \leqslant x) = \begin{cases} 0 & x \leqslant 3, \\ ax^2 - 8ax + b & 3 \leqslant x \leqslant 4, \\ 1 & 4 \leqslant x, \end{cases}$$

where a and b are constants. Show that $a = -1$, find the value of b, and verify that $P(X \leqslant 3{\cdot}5) = \frac{3}{4}$. [5]

(i) Find the probability density function of X. [2]

(ii) Show that $E(X) = \frac{10}{3}$. [3]

(iii) Three independent observations of X are taken. Find the probability that two of the observations are less than 3·5 and one is greater than 3·5. [2]

(iv) One hundred independent observations of X are taken and M is the arithmetic mean of the observations. Given that $\text{Var}(X) = \frac{1}{18}$, state the approximate distribution of M. [2]

8 [In this question give your answers correct to 3 decimal places.]

The table below gives the probability of the successful firing of randomly chosen emergency flares of various ages.

Age	1 year	5 years	10 years
Probability	0·995	0·970	0·750

(i) Using a suitable approximation, find the probability that, of 1000 randomly chosen 1-year old flares, at most 4 fail to fire successfully. [5]

(ii) Find the probability that, of 6 randomly chosen 10-year old flares, at least 3 fire successfully. [4]

(iii) Seven flares are chosen at random, of which 1 is 5 years old and 6 are 10 years old. Find the probability that

(a) the 5-year old flare fires successfully and at least 3 of the 10-year old flares fire successfully, [1]

(b) the 5-year old flare fails to fire successfully and at least 4 of the 10-year old flares fire successfully, [2]

(c) at least 4 of the 7 flares fire successfully. [2]

9 Small packets of nails are advertised as having average weight 500 g, and large packets as having average weight 1000 g. Assume that the packet weights are normally distributed with means as advertised, and standard deviations of 10 g for a small packet and 15 g for a large packet. Giving your answers correct to 3 decimal places,

(i) find the probability that a randomly chosen small packet has a weight between 495 g and 510 g, [3]

(ii) find the probability that two randomly chosen small packets have a total weight between 990 g and 1020 g, [3]

(iii) find the probability that the weight of one randomly chosen large packet exceeds the total weight of two randomly chosen small packets by at least 25 g, [4]

(iv) find the probability that one half of the weight of one randomly chosen large packet exceeds the weight of one randomly chosen small packet by at least 12·5 g. [4]

10 Cartons of milk are tested by a consumer association both for quantity and for ease of opening. A random sample of 100 cartons is examined and the quantity x litres of milk in each carton is determined. The results are summarised by $\Sigma(x_i - 1) = 1{\cdot}21$ and $\Sigma(x_i - 1)^2 = 0{\cdot}5377$. Test, at the 5% significance level, whether the mean quantity of milk in a carton is 1·005 litres, against the alternative hypothesis that it is greater than 1·005 litres. [7]

(i) It is found that 65 of the 100 cartons are easy to open. Find an approximate 90% confidence interval for the population proportion of cartons that are easy to open. [4]

(ii) A cheaper design of carton is introduced and the consumer association decides to carry out a new test. A random sample of 100 cartons is tested and 53 are found to be easy to open. Test, at the 5% significance level, whether the proportion of cartons that are easy to open is 65% or less than 65%. [3]

Option (c): Pure Mathematics

11 A curve C has equation, in polar coordinates,

$$r = a\sqrt{\{(4 + \sin^2\theta)\cos\theta\}}, \qquad (-\tfrac{1}{2}\pi \leqslant \theta \leqslant \tfrac{1}{2}\pi),$$

where a is a positive constant. Show that

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\{(4 + \sin^2\theta)\cos\theta\} = -(2 + 3\sin^2\theta)\sin\theta,$$

and hence state, with a reason, whether r increases or decreases as θ increases, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. [4]

Sketch the curve C. [3]

Making the substitution $z = \sin\theta$, show that the area A of the region enclosed by C is given by

$$A = \tfrac{1}{2}a^2 \int_{-1}^{1} (4 + z^2)\,\mathrm{d}z,$$

and hence find A. [4]

Find the cartesian equation of C in the form $(x^2 + y^2)^m = a^2x(bx^2 + cy^2)$, giving the numerical values of m, b and c. [3]

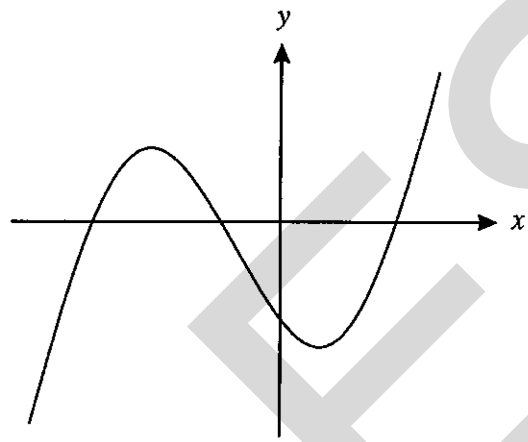
12 **(a)** The curve C has equation

$$y = -\frac{b(x+3a)}{x+a},$$

where a and b are positive constants. State, in terms of a and b, the coordinates of the points where C intersects the axes, and the equations of the asymptotes. [4]

Sketch the curve C, showing the asymptotes. [3]

(b)



A sketch of the graph of $y = \mathrm{f}(x)$ is shown in the diagram above. On separate diagrams, sketch the graphs of

(i) $y^2 = \mathrm{f}(x)$, [3]

(ii) $y = \sqrt{\{\mathrm{f}(x)\}}$, [2]

(iii) $y = |\mathrm{f}(x)|$. [2]

13 **(a)** The complex numbers z and w are such that

$$w = 1 + \mathrm{i}a, \quad z = -b - \mathrm{i},$$

where a and b are real and positive. Given that $wz = 3 - 4\mathrm{i}$, find the exact values of a and b. [7]

(b) The complex numbers z and w are such that

$$|z| = 2, \quad \arg(z) = -\tfrac{2}{3}\pi, \quad |w| = 5, \quad \arg(w) = \tfrac{3}{4}\pi.$$

Find the exact values of

(i) the real part of z and the imaginary part of z, [3]

(ii) the modulus and argument of $\dfrac{w}{z^2}$. [4]

14 **(a)** Solve the differential equation

$$\frac{d^2x}{dt^2} + 9x = 3t,$$

given that $x = 0$ and $\frac{dx}{dt} = 1$ when $t = 0$. [7]

(b) The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{2x + y + 2}{2x + y - 1}. \qquad \text{(I)}$$

Using the substitution $z = 2x + y$, show that

$$\frac{dz}{dx} = \frac{3z}{z - 1}.$$ [3]

Solve this equation and hence find the general solution of (I). [4]

15 **(a)** A curve C has equation $y = x^5 + 50x$. Find the least value of $\frac{dy}{dx}$ and hence give a reason why the equation $x^5 + 50x = 10^5$ has exactly one real root. [2]

Use the Newton-Raphson method, with a suitable first approximation, to find, correct to 4 decimal places, the root of the equation $x^5 + 50x = 10^5$. You should demonstrate that your answer has the required accuracy. [5]

(b) The equations of two planes Π_1 and Π_2 are $\mathbf{r}.(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 2$ and $\mathbf{r}.(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}) = 3$, respectively.

(i) Find the acute angle between the two planes, giving your answer to the nearest 0·1°. [3]

(ii) Find the length of the projection of the vector $\mathbf{i} + 4\mathbf{j}$ onto Π_1. [4]